

Gram-Schmidt Orthogonalization Exercises

Question 1 Perform the Gram-Schmidt Process on the following sets of vectors and also give the associated QR factorization.

$$\text{a) } \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

$$\text{c) } \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 9 \\ -5 \\ 3 \end{bmatrix}$$

$$\text{b) } \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 18 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{d) } \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \\ -1 \end{bmatrix}$$

Question 2 Find the volumes of the parallelepipeds spanned by the vectors in parts a)-d) of question 1.

Answers:

$$1\text{a) } \begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \end{bmatrix}, \frac{1}{\sqrt{18}} \begin{bmatrix} -1 \\ -1 \\ 4 \end{bmatrix},$$

$$1\text{b) } \begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \end{bmatrix}, \begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}, \begin{bmatrix} 1/3 \\ -2/3 \\ 2/3 \end{bmatrix},$$

$$1\text{c) } \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}, \begin{bmatrix} -1/10 \\ 7/10 \\ -7/10 \\ 1/10 \end{bmatrix},$$

$$1\text{d) } \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}, \begin{bmatrix} 1/2 \\ -1/2 \\ -1/2 \\ 1/2 \end{bmatrix}, \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{bmatrix},$$

$$\begin{bmatrix} 2/3 & -1/\sqrt{18} \\ 2/3 & -1/\sqrt{18} \\ 1/3 & 4/\sqrt{18} \end{bmatrix} \begin{bmatrix} 3 & 3 \\ 0 & \sqrt{18} \end{bmatrix}$$

$$\frac{1}{3} \begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 & 12 \\ 0 & 3 & -12 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & -1/10 \\ 1/2 & 7/10 \\ 1/2 & -7/10 \\ 1/2 & 1/10 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 0 & 10 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$2\text{a) } 3\sqrt{18} \quad 2\text{b) } 54 \quad 2\text{c) } 20 \quad 2\text{d) } 2$$